

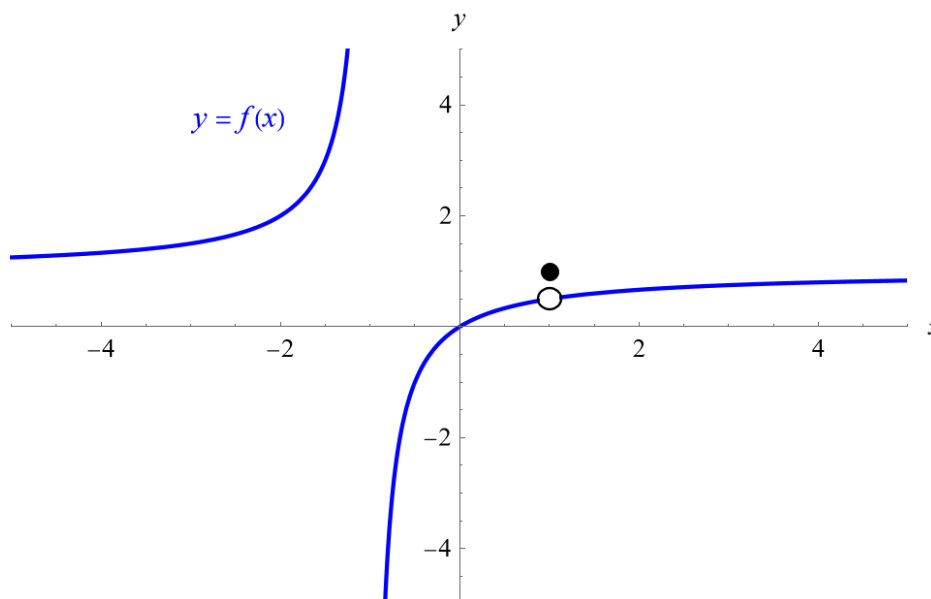
## Exercise 20

Explain why the function is discontinuous at the given number  $a$ . Sketch the graph of the function.

$$f(x) = \begin{cases} \frac{x^2 - x}{x^2 - 1} & \text{if } x \neq 1 \\ 1 & \text{if } x = 1 \end{cases} \quad a = 1$$

### Solution

A graph of the function versus  $x$  is shown below.



The function is discontinuous at  $x = 1$  because although the left-hand and right-hand limits are both equal to  $1/2$  there, they are not equal to the value of the function there, which is  $1$ .

$$\begin{aligned} \lim_{x \rightarrow 1} f(x) &= \lim_{x \rightarrow 1} \frac{x^2 - x}{x^2 - 1} \\ &= \lim_{x \rightarrow 1} \frac{x(x - 1)}{(x + 1)(x - 1)} \\ &= \lim_{x \rightarrow 1} \frac{x}{x + 1} \\ &= \frac{1}{1 + 1} \\ &= \frac{1}{2} \\ &\neq f(1) = 1 \end{aligned}$$